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U. S. FOREST SERVICE

Pacific North West Research Note

FOREST AND RANGE EXPERIMENT STATION | U.S. DEPARTMENT OF AGRICULTURE · PORTLAND, OREGON

PNW-2 May 1963

BOARD-FOOT TREE VOLUME EQUATIONS
FOR
ELECTRONIC COMPUTERS

by

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Now that electronic computers are taking over the job of processing timber cruise data, tree volume tables are being replaced by equations.

Tables can be used in electronic data processing, but they are unwieldy and they often consume too much of a computer's capacity.

Of the six equations presented in this report, two (equations 3 and 6) have been used for several years by three Federal agencies in the Pacific Northwest (Forest Service, Bureau of Land Management, Bureau of Indian Affairs) for their extensive, allowable cut surveys. Equations 1, 2, 4, and 5, which are all based on the International rule, were developed very recently for use by these agencies.

The six equations offered here are for the benefit of any other agency that might find them useful and also for the benefit of anyone interested in allowable cuts as estimated for public forests.

These equations cover all coniferous species in the Pacific Northwest. They are based on Bruce's[1] adaptation of Behre's[2] expression for tree taper. Bruce[1] invented the basic procedure for developing the equations, but he published only one of them (equation 3A below). His concern was obviously with volume tables rather than with volume equations.

Attention is directed to the verbal sections of three publications by Bruce and Girard.[3] Underlying assumptions and fundamental details of procedure for developing the equations are given in these sections. Additional details of procedure appropriate to each equation are given in the final section of this report.

All six equations require the same three tree measurements: diameter at breast height, Girard 16-foot form class, and total height from stump to tip.

The following key will assist in selecting an appropriate equation.

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[1] Bruce, Donald, and Girard, James W. Tables for estimating board foot volume of trees scaled in 16 foot logs based on diameter form class and total height. 46 pp. Portland, Oreg.: Mason, Bruce & Girard. [n.d.]

[2] Behre, C. Edward. Form-class taper curves and volume tables and their application. Jour. Agr. Res. 35: 673-744, illus. 1927.

[3] Girard, James W., and Bruce, Donald. Tables for estimating board foot volume of trees in 16 foot logs. 44 pp., illus. Portland, Oreg.: Mason, Bruce & Girard. [n.d.]

Girard, James W., and Bruce, Donald. Tables for estimating board foot volume of trees in 32 foot logs. 40 pp., illus. Portland, Oreg.: Mason, Bruce & Girard. [n.d.]

See also footnote 1.

Key to the Equations

Specification	Equation number					
	1	2	3	4	5	6
International 1/8-inch log rule	X			X		
International 1/4-inch log rule		X			X	
Formula Scribner rule[4]			X			X
Top diameter, 5 inches	X	X				
Top diameter, one-half of the diameter at top of first 16-foot log but not less than 8 inches			X	X	X	X
16-foot scaling length for logs	X	X	X	X	X	
32-foot scaling length for logs						X
0.3-foot trim allowance per log	X	X	X	X	X	
0.6-foot trim allowance per log						X

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[4] Staebler, George R. The formula Scribner log rule. U.S. Forest Serv. Pac. NW. Forest & Range Expt. Sta. Res. Note 78, 6 pp. 1952.

Special attention is directed to equation 6. The 16-foot form class is used for this equation even though the scaling length for logs is 32 feet.

Glossary of Symbols

d_i = diameter inside bark in inches at the top of the ith log in a tree

d_u = diameter inside bark in inches at the upper limit of merchantability (i.e., at the top of the top log)

d_o = diameter outside bark in inches at breast height (4.5 feet)

G = Girard form class (d_1/d_o)

H = total height of tree in feet from stump to tip

N = number of 16-foot logs in a tree

D_i = d_i/d_1 = diameter at top of the ith 16-foot log divided by diameter at top of first log

D_u = d_u/d_1

L_i = distance from tip of tree to top of ith 16-foot log divided by the distance from tip to top of first log

L_u = distance from tip of tree to top of top log divided by the distance from tip to top of first log

k = log position (i.e., $k = 2$ indicates second log from bottom and $k = N$ indicates the top log)

v_i = volume of ith log as calculated under the specifications of a particular equation

V_j = volume of a tree as calculated under the specifications of the jth equation

f = Bruce's factor for calculating tree volumes based on total height when d_1 is less than 16 inches

a_1, a_2, b_1, b_2, c_2, a_3, b_3, and c_3 = constants

Equation 1

$$V_1 = 0.0363(Gd_o)^2 + 1.0955(Gd_o) - 2.0259 + \left[0.02437(Gd_o)^2 - 0.0574(Gd_o) - 0.1306\right] H$$

Unit of measure	Board foot
Log rule	International 1/8-inch
Top diameter (d_u)	5.0 inches
Stump height	Local practice
Trim allowance per log	0.3 foot
Scaling length of logs	16 feet

Equation 2

$$V_2 = 0.0328(Gd_o)^2 + 0.9912(Gd_o) - 1.8330 + \left[0.02205(Gd_o)^2 - 0.0519(Gd_o) - 0.1182\right] H$$

Unit of measure	Board foot
Log rule	International 1/4-inch
Top diameter (d_u)	5.0 inches
Stump height	Local practice
Trim allowance per log	0.3 foot
Scaling length of logs	16 feet

Equations 3A and 3B

$Gd_o \geq 16$

$$V_{3A} = \left[0.02032(Gd_o)^2 - 0.0637(Gd_o) - 0.1625\right] (H+8)$$

$Gd_o < 16$

$$V_{3B} = 30 + \left[0.01610(Gd_o)^2 + 0.1118(Gd_o) - 1.8893\right] H$$

Unit of measure	Board foot
Log rule	Scribner formula
Top diameter (d_u)	50 percent of d_1, but not less than 8.0 inches

Equations 3A and 3B (continued)

Stump height	Local practice
Trim allowance per log	0.3 foot
Scaling length of logs	16 feet

Equations 4A and 4B

$Gd_o \geq 16.00$ inches

$$V_{4A} = \left[0.02264(Gd_o)^2 - 0.0484(Gd_o) - 0.0552\right](H+8)$$

$Gd_o < 16.00$ inches

$$V_{4B} = 38 + \left[0.01596(Gd_o)^2 + 0.2239(Gd_o) - 2.6898\right] H$$

Unit of measure	Board foot
Log rule	International 1/8-inch
Top diameter (d_u)	50 percent of d_1, but not less than 8.0 inches
Stump height	Local practice
Trim allowance per log	0.3 foot
Scaling length of logs	16 feet

Equations 5A and 5B

$Gd_o \geq 16.00$ inches

$$V_{5A} = \left[0.02048(Gd_o)^2 - 0.0438(Gd_o) - 0.0499\right](H+8)$$

$Gd_o < 16.00$ inches

$$V_{5B} = 34 + \left[0.01444(Gd_o)^2 + 0.2026(Gd_o) - 2.4336\right] H$$

Unit of measure	Board foot
Log rule	International 1/4-inch
Top diameter (d_u)	50 percent of d_1, but not less than 8.0 inches
Stump height	Local practice
Trim allowance per log	0.3 foot
Scaling length of logs	16 feet

Equations 6A, 6B, and 6C

$Gd_o \geq 14.4$

$$V_{6A} = -0.2541(Gd_o)^2 + 0.2894(Gd_o) - 1.3600 + \left[0.02064(Gd_o)^2 - 0.0643(Gd_o) - 0.1624\right] H$$

$13.5 \leq Gd_o < 14.4$

$$V_{6B} = -50 + \left[0.02064(Gd_o)^2 - 0.0643(Gd_o) - 0.1624\right] H$$

$Gd_o < 13.5$

$$V_{6C} = -15.3180(Gd_o) + 156.8560 + \left[0.5214(Gd_o) - 4.3340\right] H$$

Unit of measure ----------------------------	Board foot
Log rule ------------------------------------	Scribner formula
Top d. i. b. (d_u) ---------------------------	50 percent of d_1, but not less than 8.0 inches
Stump height --------------------------------	Local practice
Trim allowance per log ----------------------	0.6 foot
Scaling length of logs -----------------------	32 feet

Procedure Used in Developing the Equations

Equation 1

Step 1.--For a hypothetical tree with particular values for d_1 and N, 16-foot scaling diameters for each log in the tree were calculated by:

$$d_i = d_1 D_i \text{ ---------------------- } (1)$$

where

$$D_i = \frac{L_i}{0.49\ L_i + 0.51} \text{ ------------ } (2)$$

$$L_i = \frac{H - (16.3)k}{H - 16.3} \text{ -------------- } (3)$$

$$k = 2, 3, 4 \ldots . (N-1) \text{ ------------- } (4)$$

$$L_u = \frac{H - (16.3)N}{H - 16.3} \text{ ----------------- } (5)$$

$$H = \frac{(N - L_u)16.3}{1 - L_u} \text{ ----------------- } (6)$$

$$L_u = \frac{D_u(0.51)}{1 - D_u(0.49)} \text{ ----------------- } (7)$$

$$D_u = \frac{d_u}{d_1} \text{ ----------------------- } (8)$$

$$d_u = 5 \text{ ------------------------- } (9)$$

Step 2.-- For each scaling diameter calculated in step 1, log volume according to the International 1/8-inch rule was calculated by:

$$v_i = 0.88d_i^2 - 1.52d_i - 1.36$$

Step 3.-- Volumes for all logs in the hypothetical tree were then added.

Step 4.-- Steps 1, 2, and 3 were repeated for several hypothetical trees, all of which had the same d_1, but with N different for each tree.

Step 5.--Constants a_1 and b_1 for the relationship $V_1 = a_1 + b_1H$ were calculated using the several tree volumes from step 4. Note that N can be converted to H from (6). There was no residual variation in the relationship V on H.

Step 6.-- Steps 1 through 5 were repeated for several different values of d_1.

Step 7.--Constants a_2, b_2, and c_2 were calculated for the following curvilinear relationship of b_1 on d_1:

$$b_1 = a_2 + b_2 d_1 + c_2 d_1^2$$

Step 8.--Constants a_3, b_3, and c_3 were calculated for this curvilinear relationship of a_1 on d_1:

$$a_1 = a_3 + b_3 d_1 + c_3 d_1^2$$

Step 9.--Results of steps 7 and 8 led directly to equation 1:

$$V_1 = a_3 + b_3 d_1 + c_3 d_1^2 + \left[a_2 + b_2 d_1 + c_2 d_1^2\right] H$$

Equation 2

Each constant in the V_1 equation was simply multiplied by 0.904762 which is the factor for converting volume by the International 1/8-inch rule to volume by the International 1/4-inch rule.

Equation 3A

This equation has already been presented in the Bruce-Girard green book,[5] and the procedure used in its development has been described on pages 41-46 of that publication.

Equation 3B

Equation 3A is applicable only when the diameter at the limit of merchantability is 50 percent of d_1. However, when a minimum 8-inch top merchantability limit is imposed, equation 3A will not be appropriate. This will be true whenever d_1 is less than 16 inches.

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[5] See footnote 1.

Bruce[6] described the procedure for the case where d_1 is less than 16 inches (pages 45-46 in his green book), but he gave no equation. Instead, he gave a table of factors which, when multiplied by H, would give tree volume less 30 board feet.

These factors could be used directly in electronic computers. However, since an equation may be preferred, constants for a second degree curve were found for the relation between Bruce's factors and d_1. Thus:

$$f = a + bd_1 + cd_1^2$$

where f = Bruce's factor

and since

$$V_{3B} = fH + 30$$

$$V_{3B} = \left[a + bd_1 + cd_1^2 \right] H + 30$$

Equation 4A

Specifications for tree volumes under equation 4A are identical with specifications for equation 3A with the one exception that the International 1/8-inch rule was used for 4A while the formula Scribner rule was used for 3A.

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[6] See footnote 1.

For the International 1/8-inch rule:

$$v_i = 0.88d_i^2 - 1.52d_i - 1.36$$

and for the formula Scribner rule:

$$v_i = 0.79d_i^2 - 2d_i - 4$$

Constants for equation 4A were found by applying ratios of corresponding constants in the log rules to constants in equation 3A. Thus:

$$0.02264 = 0.02032\left(\frac{0.88}{0.79}\right)$$

$$0.0484 = 0.0637\left(\frac{1.52}{2}\right)$$

$$0.0552 = 0.1625\left(\frac{1.36}{4}\right)$$

Equation 4B

The procedure used for equation 4B was identical to the procedure used for equation 1 with these exceptions:

1. $D_u = 8/d_1$
2. Calculations were limited to values of d_1 less than 16.
3. Values for a_1 (steps 5 and 8 under Procedure for Equation 1) were all so close to 38 that 38 could safely be substituted for the first three terms in equation 1.

Equations 5A and 5B

Constants in equations 4A and 4B were simply multiplied by 0.904762 to give corresponding constants for 5A and 5B.

Equation 6A

The procedure used for equation 1 was also used for equation 6A. The exceptions in this case were:

1. $D_u = \frac{d_u}{d_l} = 0.5$

2. Scaling diameters for 32-foot logs instead of for 16-foot logs were found in step 1. (Note that d_l in this case was still taken as diameter at the top of the first 16-foot log even though logs were scaled in 32-foot lengths.)

3. Although equation 6A was developed for the case where d_l was 16 inches or greater, it was found that by applying it down to $d_l = 14.4$, a better transition to equation 6B was effected.

Equation 6B

Equation 6A was originally extended down to $d_l = 13.5$, but it was found that if the sum of the first three terms of 6A were taken as (-50) for all cases where $13.5 \leq d_l < 14.4$, a better transition was effected between 6A and 6C.

Equation 6C

The general procedure used for equation 1 was also used here with these exceptions:

1. $D_u = 8/d_l$

2. Scaling diameters for 32-foot logs instead of for 16-foot logs were found in step 1. (Note that d_l is still the diameter at the top of the first 16-foot log and that the 16-foot form class is implied.)

3. Linear instead of curvilinear relationships were found for b_1 on d_1 and for a_1 on d_1 in steps 7 and 8. Thus the c_2 and c_3 constants were not needed. In this restricted range of d_1's (i.e., 8.0 inches to 13.5 inches), there was no evidence of curvilinearity.